Exercise 4.3

Q.1. Find the roots of the following quadratic equations, if they exist, by the method of completing the square.

(i) $2x^2 - 7x + 3 = 0$ (ii) $2x^2 + x - 4 = 0$

(iii) $4x^2 + 4 + 3 = 0$ (iv) $2x^2 + x + 4 = 0$

Sol.   (i) $2x^2 - 7x + 3 = 0$

$$x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

$$\Rightarrow \left\{ x - \frac{1}{2}\left(\frac{7}{2}\right)\right\}^2 - \left(\frac{7}{4}\right)^2 + \frac{3}{2} = 0$$

$$\Rightarrow \left\{ x - \frac{7}{4}\right\}^2 - \frac{49}{16} + \frac{3}{2} = 0$$

$$\Rightarrow \left\{ x - \frac{7}{4}\right\}^2 - \frac{49}{16} + \frac{24}{16} = 0$$

$$\Rightarrow \left\{ x - \frac{7}{4}\right\}^2 - \frac{25}{16} = 0$$

$$\Rightarrow \left\{ x - \frac{7}{4}\right\}^2 = \frac{25}{16} = \left(\frac{5}{4}\right)^2$$

$$\Rightarrow x - \frac{7}{4} = \pm\frac{5}{4}$$

Case I:

When $\frac{5}{4}$ is +ve, then

$$x - \frac{7}{4} = \frac{5}{4} \Rightarrow x = \frac{5}{4} + \frac{7}{4}$$

$$\Rightarrow x = \frac{12}{4} = 3$$

Case II:

When $\frac{5}{4}$ is +ve, then

$$x - \frac{7}{4} = \frac{5}{4} \Rightarrow x = \frac{5}{4} + \frac{7}{4}$$

$$\Rightarrow x = \frac{12}{4} = 3$$

Case II:

When $\frac{5}{4}$ is +ve, then

$$x - \frac{7}{4} = -\frac{5}{4} \Rightarrow x = -\frac{5}{4} + \frac{7}{4}$$

$$\Rightarrow x = \frac{2}{4} = \frac{1}{2}$$

Thus, required roots are

$$x = 3 \text{ and } x = \frac{1}{2}$$

Q.2. Find the roots of the following quadratic equations, using the quadratic formula:

(i) $2x^2 - 7x + 3 = 0$

(ii) $2x^2 + x - 4 = 0$

(iii) (iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

(iv) $2x^2 + x + 4 = 0$

Sol.   (i) $2x^2 - 7x + 3 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have

$a = 2$

$b = -7$

$c = 3$

$\therefore b^2 - 4ac = (-7)^2 - 4\,(2)(3)$

$= 49 - 24 = 25\ 0$

Since $b^2 - 4ac > 0$

$\therefore$ The given equation has real roots. The roots are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-(-7) \pm \sqrt{25}}{2(2)}$$

$$= \frac{7 \pm 5}{4}$$

Taking +ve sign,

$$x = \frac{7+5}{4} = \frac{12}{4} = 3$$

Taking –ve sign,

$$x = \frac{7-5}{4} = \frac{2}{4} = \frac{1}{2}$$

Thus, the roots of the given equation are

$$x = 3 \text{ and } x = \frac{1}{2}$$